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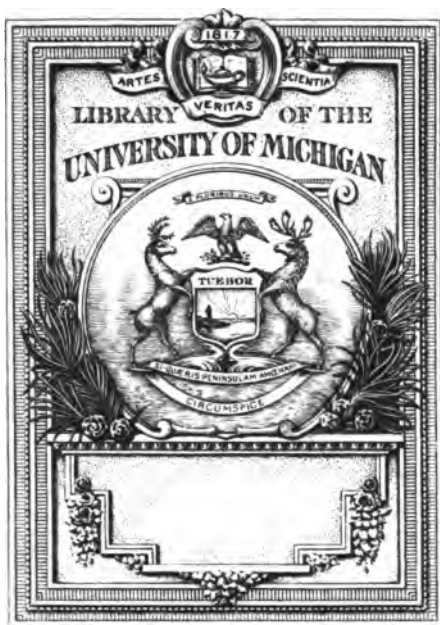
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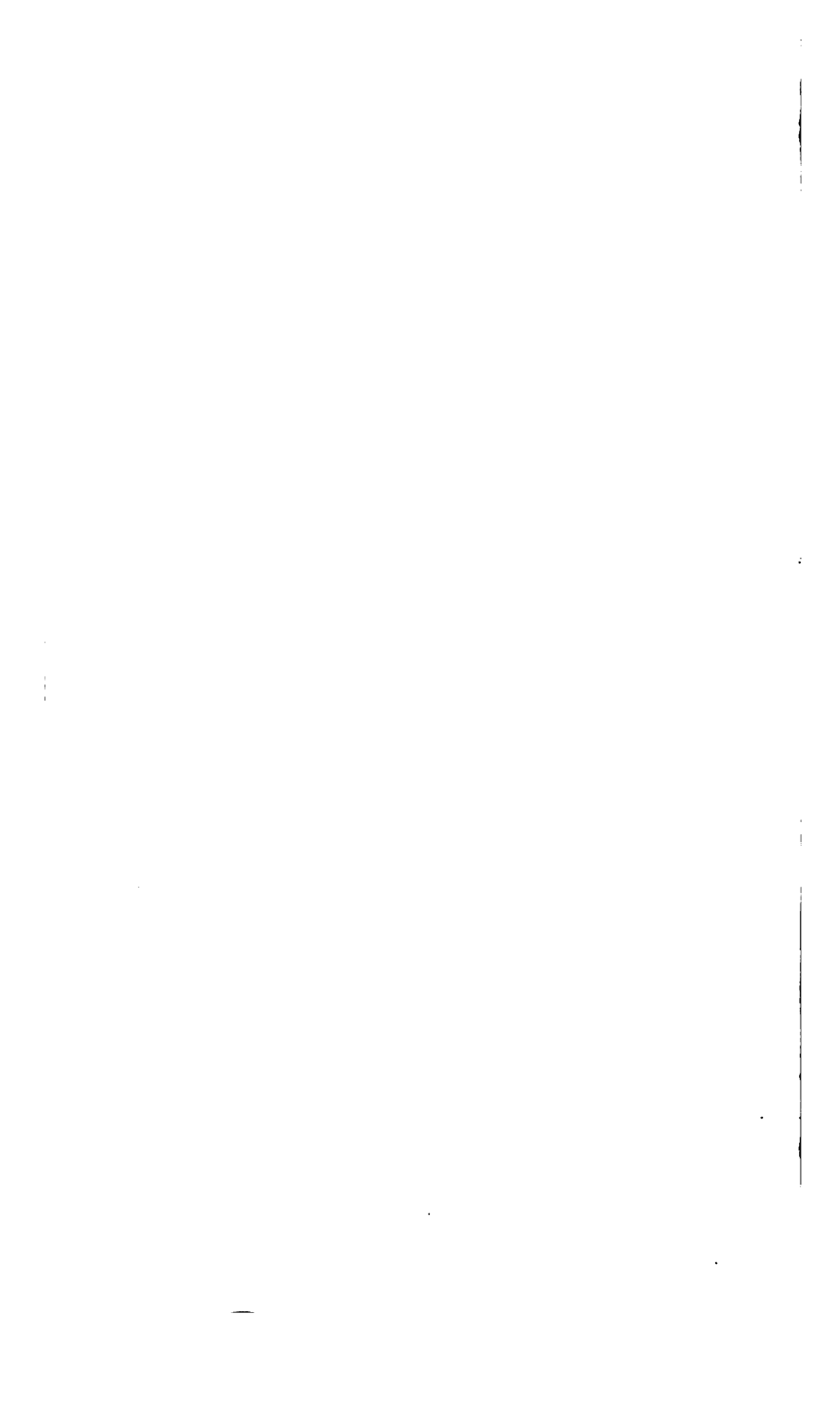
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DISSERTATION
ON THE
GEOMETRICAL ANALYSIS
OF THE
ANTIENTS.

WITH A
Collection of THEOREMS and PROBLEMS,
WITHOUT SOLUTIONS,
FOR THE
EXERCISE of YOUNG STUDENTS.

By
John Lawson, B.D. Rector of Swanscombe in Kent.

PUNGAR VICE COTIS.

CANTERBURY,

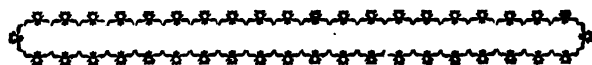
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A
DISSERTATION
ON THE
GEOMETRICAL ANALYSIS
OF THE
A N T I E N T S.



IT is universally allowed that Mathematical Studies are attended with much pleasure and delight, and that they are in a particular manner captivating and engaging to those whose genius and bent of mind lie that way; and it is equally certain that they are profitable as well as pleasant. From the proper cultivation of them many emoluments accrue to society, and many improvements are made in civil life. But setting aside these public advantages, for which the world must be indebted to men of superior ability and exalted genius, qualified by Nature to extend the bounds of Science, and to derive

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practical improvements from their abstract reasonings ; I would now only recommend these studies, as useful to men of all ranks, professions and abilities, in their private capacities. They help greatly to fix the attention, to enlarge the understanding, to methodise our ideas, and to improve our reasoning faculties, when we apply them to any other science whatsoever. Great regard was had to these sciences by the ancients in their education of children ; and they ought certainly to have a principal share in the education of gentlemen and scholars, if we would have them taught to think justly or to reason well. Of all the branches of Mathematical Science none conduces more to the great ends here mentioned than Geometry. The simplicity of it's first principles, the clearness and evidence of it's demonstrations, the admirable concatenation of it's parts, and regular connexion of the propositions with one another, tend greatly to establish a habit of close thinking, and a methodical and just argumentation, when we apply ourselves to any other subject whatsoever, physical or moral, oeconomical or civil.

But

But I know not how it happens, this noble science seems of late to have met with less regard than it's dignity and usefulness demand. Our schools and universities*, our philosophical societies, and philomaths of all degrees, seem to have been very assiduous of late in paying their devoirs to the younger sister *Algebra*; and at the same time to have overlooked, and in a great measure neglected, those native charms, that amiable simplicity, and those more attractive excellencies, that peculiarly belong to the elder. I would not be thought any ways to disparage the *algebraical* art, or to derogate from it's merits; they are doubtless very great. All the branches of the Mathematics are much indebted thereto; and even Geometry itself has within this last century been surprisngly extended and improved thereby. But, notwithstanding this, I must affirm that, for those who apply themselves to the Mathematics only for the ends above-mentioned, namely, to inure themselves to a method of close thinking and

* The reader is referred to some late resolutions taken in the university of Cambridge.

just reasoning, *Geometry* is the proper field : and that those who but moderately exercise themselves therein will sooner attain their purpose, than others that may go far greater lengths in the intricacies of *Algebra*, and the labyrinth of *Fluxions*.

It is but too common a practice with young students, after having gone through the Elements of *Euclid* in a cursory manner, (or perhaps having substituted in their place some other *superficial* and *less geometrical* elements,) from henceforth to bid adieu to all demonstrations strictly geometrical, and to employ themselves in the consideration of algebraical equations, infinite series, the algorithm of fluxions, the properties of curve lines, and other parts of the *higher* Geometry, when perhaps they are but very superficially acquainted with the first elements of the *plane*. If they read a system of conics, they will be sure to make choice of one, where the demonstrations are algebraical ; and when they apply themselves to the solution of geometrical problems, here again they will have recourse to
their

their Algebra, not at all apprized perhaps of any other analysis; being utterly unacquainted with the method of resolution and composition so carefully cultivated by the antient geometers. Such students may become dextrous calculators, but such a method of proceeding tends very little towards bettering the judgment, or improving the invention.

The study of Geometry I say is the most proper for young men to pursue, in order to acquire a vigorous constitution of mind, and is as conducive thereto as exercise is towards procuring health and strength to the body. *Logical* precepts are useful, and indeed necessary for those that are engaged in public disputations, or controversial writings, in order to put to silence an obstinate adversary. But ‘in the search of truth, an imitation of the method of *Geometers* will carry a man further than all the *dialectical* rules. Their *Analysis* is the proper model we ought to form ourselves upon, and imitate in the regular disposition, and gradual progress of our enquiries*.’

* *Essay on the usefulness of Mathematical Learning. Oxf. 1701.*

We are told by Dr. *Pemberton**, ‘ that Sir
 ‘ *Isaac Newton* used to censure himself for not
 ‘ following the Antients more closely than he
 ‘ did ; and spoke with regret of his mistake,
 ‘ at the beginning of his mathematical studies,
 ‘ in applying himself to the works of *Des-*
 ‘ *cartes*, and other algebraical writers, before
 ‘ he had considered the Elements of *Euclid* with
 ‘ that attention so excellent a writer deserves.
 ‘ That he highly approved the laudable at-
 ‘ tempt of *Hugo de Omerique* to restore the
 ‘ *antient Analysis*.’ Now what the great Sir
Isaac Newton so highly approved, it is the in-
 tention of this publication more particularly to
 specify and recommend. Little has yet been
 done toward the attainment of this laudable
 purpose of restoring the antient Analysis. The
 writer just mentioned is very little known in
 England. The Author of this small Tract is
 willing to contribute his mite, and very de-
 sirous to revive a proper taste for pure Geometry.
 He has annexed a collection of *Theorems* and
 likewise a few *Problems*, to be solved by the

* In the preface to his *View*, &c.

Geometrical Analysis: he has been more sparing in the latter, because plenty of them are continually proposed in periodical publications. It is not pretended that they are new ones; but they are such as rarely occur to them for whose use they are principally intended. Not above four or five of them, I believe, have ever appeared in English before; and they are all taken from authors which seldom fall into the hands of young men. They will serve therefore as proper Exercises for young students to try their strength upon.

But before they set themselves to this work, I would recommend a very careful and reiterated perusal of the Elements, and after that as diligent an application to that valuable remains of antiquity, the book of *Euclid's Data*, both which they will find most compleat in Dr. *Robert Simson's* edition. When they have made themselves perfect masters of these, they may then betake themselves to the solution of Geometrical Propositions by a Geometrical Analysis; either that of the Antients derived from the Data: or, if this should be thought too tedious

tedious and troublesome, they may abate somewhat of it's rigour, and still make use of a similar method : but I would have them by no means content themselves with algebraical resolutions, even though they should be able to derive constructions from thence, and also to demonstrate synthetically the truth of the same. How they may proceed with success I shall endeavour briefly to explain.

Resolution then or *Analysis* is the method of proceeding from the thing *sought* as *taken* for granted through it's consequences to something that is *really* granted ; and *Composition* or *Synthesis* is a reverse method, wherein we lay that down first which was the last step of the *Analysis*, and tracing the steps of the *Analysis* back, making that antecedent here which was consequent there, till we arrive at the thing *sought*, which was put as granted in the first step of the *Analysis*.

When we are to apply this method of *Resolution* to *Theorems*, we must first lay what is therein affirmed down as true, and then consider

consider the necessary consequences flowing therefrom, deducing one consequence from another, till we arrive at last at some one, which is evidently true or evidently false; as may appear by an Axiom, or an Elementary Proposition, or by what is called *Exposition*, i. e. the nature and structure of the figure. When the former is the case, the *Theorem* is true and may be demonstrated by the method of *Composition*, but when the latter is the case, it is false, for all truths are consistent with each other. An example will clear this more than many words.

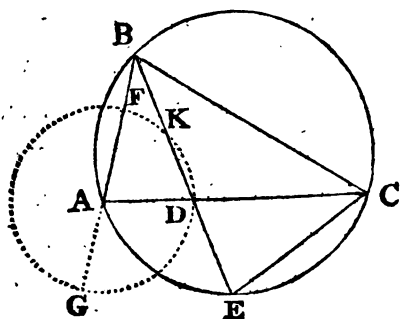
THEOREM.

The square of a line bisecting the vertical angle of any triangle, together with the rectangle under the segments of the base made thereby, is equal to the rectangle under the sides containing that angle.

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ANALYSIS.

(x)



ANALYSIS.

Suppose this to be true, viz. that $BD^2 + ADC = AB \times BC$, and let a circle be circumscribed about the triangle, and BD produced to meet it in E, and EC joined. Now $ADC = BDE$ by Euc. III. 35. Therefore $BD^2 + ADC = BD^2 + BDE = EBD$ by II. 3. Therefore also $AB \times BC = EBD$. Now this we shall find to be true by the Elements, hence the theorem is also true. For the triangles ABD and EBC are similar, having the angles at A and E equal as standing on the same circumference and the angles at B in each equal by *Exposition*, therefore by VI. 4. $AB : BD :: BE : BC$ and by VI. 16. $AB \times BC = EBD$.

SYNTHESIS.

SYNTHESIS.

$$AB \times BC = EBD \text{ (as proved in the Analysis)}$$

$$= BD^2 + BDE \text{ by II. 3.} = BD^2 + ADC \text{ by III. 35.}$$

Q. E. D.

.. It will be very proper for young students to endeavour to obtain a variety of demonstrations of one and the same proposition, deduced from different principles; hereby they will be able to discover which are the most simple and elegant, and greatly improve their taste and judgment. An instance of the great fecundity of Geometry in this respect is given in the beginning of a periodical work published a few years ago under the title of the **BRITISH ORACLE**, where there are given fifteen demonstrations of one and the same Theorem, all independent of each other, and derived from very different principles and constructions. I shall now therefore proceed to give another demonstration of the foregoing theorem, not as more simple than the preceding, but as less so, and further fetched. It was however given by an author of no small note in the last century.

ANALYSIS.

With one of the angular points at the base as center, and the adjacent segment as radius, describe a circle, which let cut the adjacent side in F, and the same produced in G, and the bisecting line in K. $AB : BC :: AD : DC$ by VI. 3. and $AB : BC :: AB^2 : AB \times BC$, and $AD : DC :: AD^2 : ADC$ by VI. 1. Therefore by V. 19, $AD : DC :: AB^2 - AD^2 : AB \times BC - ADC$. Now $AB^2 - AD^2 = AB^2 - AF^2 = GBF$ by II. 6. $= DBK$ by III. 36. and $AB \times BC - ADC = BD^2$ by hypothesis (for $AB \times BC$ is put $= BD^2 + ADC$.) Therefore $AD : DC :: DBK : BD^2$. But $DBK : BD^2 :: BK : BD$ by VI. 1. Therefore $AD : DC :: BK : BD$, which is true by Theorem LVIII. in the following collection.

SYNTHESIS.

$AD : DC :: BK : BD$ (by the Theorem) $:: DBK : BD^2$ by VI. 1. Now $DBK = GBF$ by III. 36. $= AB^2 - AF^2$ by II. 6. $= AB^2 - AD^2$. Therefore $AD : DC :: AB^2 - AD^2 : BD^2$. Moreover $AD : DC :: AD^2 : ADC$ by VI. 1. Therefore by V. 12. $AD : DC :: AB^2 : BD^2 + ADC$.

ADC: Now $AD : DC :: AB : BC$ by VI. 5, and $AB : BC :: AB^2 : AB \times BC$ by VI. 1. Therefore by equality $AB^2 : BD^2 + ADC :: AB^2 : AB \times BC$, and by V. 9, $BD^2 + ADC = AB \times BC$.

Q. E. D.

• Here it is evident that this demonstration falls short of the preceding, because it does not flow immediately from an elementary proposition, but in order to its ratification the fore-cited Theorem LVIII. must be previously demonstrated as a Lemma.

When a *Problem* is proposed to be solved, we must apply our method of *Resolution* thus. We must conceive the thing required to be already done, and from this supposition we must reason, deducing one consequence from another, and proceeding step by step, till we can arrive at something that is granted, something that may be effected by means of the Postulates and Elementary Propositions, something which (in the style of the Antients) is given, or a *Datum*. Which if we can do, we shall then be able to form

form our *Synthesis* or *Composition* by making the *Datum*, we arrived at in the last step of our *Analysis*, the first step or foundation of our *Synthesis*; and then reasoning in a retrograde order, and taking the same steps back again, we shall deduce one consequence from another, till we arrive at the original *Quæsitum*, or thing required to be done in the Problem proposed, which was the first thing laid down and supposed in our *Analysis*.

Take the following example, being the 155th Proposition of *Pappus's* VIIth book.

P R O B L E M.

It is required in a given segment of a circle from the extremes of the base A and B to draw two lines AC and BC meeting at a point C in the circumference, and such that they shall have a given ratio to each other, viz. that of F to G.

ANALYSIS.

Suppose the thing done, and that the point C is found: then by way of preparation or construction, or something to found our Analysis upon, let us suppose that a tangent to the segment at the point C is drawn, which meets AB produced in the point D. Now by hypothesis $AC : CB :: F : G$, also $AC^2 : CB^2 :: AD : DB$, which is thus proved.

Since DC touches the circle and BC cuts it, the angle $DCB = BAC$ by III. 32. Also the angle D is common to both the triangles CDB and CDA, therefore they are similar, and by VI. 4. $AD : DC :: DC : DB$, hence $AD^2 : DC^2 :: AD : DB$ by VI. 20. cor. But also by VI. 4.

AD

$AD : AC :: DC : CB$, and by permutation
 $AD : DC :: AC : CB$ or $AD^2 : DC^2 :: AC^2 : CB^2$,
 therefore by equality $AC^2 : CB^2 :: AD : DB$.

But the ratio of $AC^2 : CB^2$ is given, (by Prop. LVII. in Dr. Simson's edition of the *Data**,) because the ratio of $AC : CB$ is given, therefore also that of $AD : DB$. Now since the ratio of $AD : DB$ is given, therefore also, by *Data* VI. that of $AD : AB$; and hence by *Data* II. AD is given in magnitude.

And here the Analysis properly ends. For it having been shewn that AD is given, or that a point D may be found in AB produced such, that from it a tangent being drawn to the circumference, the point of contact will be the point sought; we may now begin our Composition or Synthetical Demonstration: which we must do by finding the point D , or laying down the line AD , which we affirmed to be given in the last step of our Analysis.

* Dr. Simson has altered the Order of the Propositions of this book, but by marginal figures referred to the original order in the Greek text.

SYNTHESIS.

SYNTHESIS.

Construction. Make as $F^2 : G^2 :: AD : DB$ (which may be done, since AB is given, by making it as $F^2 - G^2 : G^2 :: AB : DB$, and then by composition it will be as $F^2 : G^2 :: AD : DB$) and then from the point D thus found draw a tangent to the circle, and from the point of contact C drawing CA and CB the thing is done.

Demonstration. Since by *construction* $F^2 : G^2 :: AD : DB$, and also $AD : DB :: AC^2 : BC^2$ (which has been already demonstrated in the analysis and may be here proved in the same manner.) Therefore $F^2 : G^2 :: AC^2 : BC^2$, and consequently $F : G :: AC : BC$.

Q. E. D.

Here we see an instance of the method of *resolution* and *composition*, as it was practised by the Antients, for the solution here given is that of *Pappus Alexandrinus*. But because the method of referring and reducing every thing to the *Data*, and constantly quoting the same, may appear to many to be very tedious and troublesome: and indeed it is unnecessary to

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those

those who have already made themselves masters of the substance of this valuable book of *Euclid*, and have by practice and experience acquired a facility of reasoning in such matters; I shall therefore now shew how we may abate something of the rigour and strict form of the antient method of demonstration, without diminishing any part of it's admirable perspicuity and elegance. And this I shall do by instancing in another solution or two of the same problem.

But before I do this, it may be proper to take notice that, in this business of the resolution of problems, every thing cannot be brought within strict rules; nor any infallible directions given, whereby a man may be enabled to succeed in all possible cases; but that there is need of a previous preparation, a kind of mental contrivance and construction, in order to form a connexion between the *Data* and *Quæsitæ*, which must be left to every one's own sagacity to find out. And it is on this very account chiefly that I recommend the exercise and employment of solving *Geometrical Problems*, as a means to help our invention, and to improve and strengthen

strengthen our reasoning powers, when we employ them on any other subject of a different nature. And here it is that the Geometrician shews his taste and acuteness in choosing the proper foundation, whereon to build the most elegant construction and demonstration; and the mathematical reader his judgment in being able, amidst a variety, to distinguish the same. However some people may be apt to ridicule the notion of taste, when applied to these subjects, yet I do maintain, that the word and thing signified thereby is very properly applicable thereto: and I believe all true mathematicians will bear me out, when I affirm that a man may shew as much taste in being able to distinguish between the different degrees of elegance in mathematical demonstrations, as a student in the *Belles Lettres*, or a professed critic, can, by his right relish for the *sublime* or *pathetic* in poetry or oratory.

But to return: before we begin our analysis, I say there is most commonly some previous preparation necessary, in order to form a connexion between the *Data* and *Quæstia*, which cannot

fall within any rules ; but is various, according to the various nature of the problems proposed. Right lines must be drawn in particular directions ; or of particular magnitudes : bisecting perhaps a given angle, or perpendicular to a given line ; tangents from a given point to a given curve ; circles must be described from a given center with a given radius ; or touching given lines or other given circles ; or such-like other operations. Whoever is conversant with the works of *Archimedes*, *Apollonius* or *Pappus*, knows very well that they all found their analysis upon such-like previous operations. Now the great skill of the Analyst consists in discovering the most proper *effections* whereon to found his Analysis : and young students ought to exercise their abilities and improve their sagacity therein : for the same problem may frequently be constructed many different ways, and many different demonstrations given of the same ; which I shall now exemplify by giving two more different solutions of the aforesaid problem.

ANALYSIS.

ANALYSIS.

Let us suppose then again that the thing is done, *i. e.* $AC : CB :: F : G$, (see the former figure) and let the base of the segment AB be cut in the same ratio in the point E. Then EC being drawn will bisect the angle ACB by VI. 3. consequently if the circle be completed and CE produced to meet it in K, the remaining circumference will also be bisected in K. Therefore the point K, as well as E, being given, the point C must also be given.

SYNTHESIS.

Construction. Let the given base of the segment AB be cut in the point E in the assigned ratio of $F : G$ by VI. 10. and complete the circle by III. 25. bisect the remaining circumference in K by III. 30. join KE and continue it to meet the given circumference in C, and drawing CA, CB, the thing is done.

Demonstration. Since the arc KC = the arc KB, the angle ACK = the angle BCK by III. 27. Therefore $AC : CB :: AE : EB$ by VI. 3.
but

but $AE : EB :: F : G$ by *construction*; therefore
 $AC : CB :: F : G$.

Q. E. D.

Again this problem may be solved in another manner, by considering it in a little different light; for it is the same thing required as, having the base of a triangle given, together with the vertical angle and the ratio of the legs, to find the triangle.

Let $P =$ the given angle, or in other words the angle in the given segment. Set off upon the legs of the angle P , PM and PQ in the assigned ratio of $F : G$, and join MQ . Then upon the given base AB make a triangle ACB similar to MPQ by VI. 18. and the thing is done; and the demonstration follows immediately from VI. 4.

Having now given a specimen of what kind of solutions I would have the young student endeavour after, I have nothing further to add, but to advise him, when he undertakes a proposition, not to be discouraged by one or two fruit-

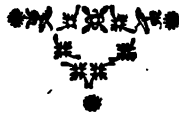
fruitless attempts, and thereupon throw it aside, or ransack books for the solution ; but if he would profit by this exercise, let him persevere, encouraged by the words of *Syrus* in the play ;

Nil tam difficile est quin quærendo investigari potest.



N. B. Of the propositions here annext for exercises, the first thirty-seven theorems have figures to explain the same, contained in two copper-plates : the rest of the theorems and the problems were purposely selected such as may be sufficiently explained by words alone, so that the reader may easily supply the figure. This was done that the expence might not exceed the pockets of those for whom these exercises are principally calculated. Several of the theorems may seem to be dependent upon some of the preceding ones, and may indeed be most commodiously deduced therefrom, which I shall here enumerate, viz. II. IV. V. VII. VIII. X. XII. XIII. XIV. XV. XVII. XX. XXII.

XXII. XXIV. XXVI. XXVII. XXVIII. and XXXV. But the demonstrations of all the rest are independent and may be derived immediately from the Elements ; as may also the demonstrations of some of those enumerated, which it will be worth the student's while to try.



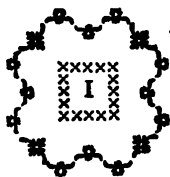
GEOME-



GEOMETRICAL THEOREMS.



P R O P . I.



F a right line AB be bisected in E ,
and two points C & D taken there-
in such that $AC:CB::AD:DB$;
then I say the rectangle $DCE =$
the rectangle ACB .

The converse of this proposition is also true,
which is this.

If a right line AB be bisected in E , and
two points C and D taken therein such that
 $DCE = ACB$; then I say $AC:CB::AD:DB$.

B

PROP. II.

P R O P. II.

If in AB the diameter of a circle two points C and D be assumed such that $AC:CB::AD:DB$, and from D an indefinite perpendicular to the same diameter as LD be erected, and through C any line be drawn to cut the same in E , and the circle in F and G ; I say $FC:CG::FE:EG$.

The converse of this proposition is also true, which is this.

If any right line as LD be drawn perpendicular to the diameter AB of any circle and meets the same in D , and if from a point in the same diameter, as C , any line be drawn to meet the same perpendicular in E , and the circle in F and G , so that $FC:CG::FE:EG$; I say that $AC:CB::AD:DB$.

P R O P. III.

Let there be a triangle ABC , whose base BC is bisected in D , and through the vertex A a line AE drawn parallel to BC , and any line drawn through D to meet AB , AC , AE in F , G , H ; then I say $GD:DF::GH:HF$.

PROP. IV.

[3]

P R O P. IV.

If in AB the diameter of a circle two points C and D be taken such that $AC:CB::AD:DB$, and through the point D any line be drawn to meet the circle in E and F, and CE, CF be joined; then I say $EC:CF::ED:DF$.

P R O P. V.

If the base BC of a triangle be bisected in D, and through the vertex A a parallel thereto be drawn, and from D a perpendicular to BC be drawn to meet the parallel in E, and through D any line be drawn to meet AB, AC in F and G, and EF, EG be joined; then I say $EF:EG::FD:DG$.

P R O P. VI.

If in the line AB be taken two points C and D such that $AC:CB::AD:DB$, and AE, BF be drawn perpendicular to AB, and through the point C be drawn any line to meet AE, BF in G and H, and DG, DH be joined; then I say that $DG:DH::GC:CH$.

P R O P. VII.

If in the diameter of a circle AB be taken any point C, and CDE be drawn meeting the circle in D and E, and DF be perpendicular to AB meeting it F, and the circle again in G, and EG be joined meeting AB in H; I say that $AC : CB :: AH : HB$.

Also, as the converse, that if in the diameter AB two points be taken as C and H such that $AC : CB :: AH : HB$, and from the points C and H two lines CE, HE be inflected to any point of the circumference E meeting the same again in D and G; when DG is drawn, it will be perpendicular to AB.

P R O P. VIII.

If in the diameter of a circle AB two points C and H be taken such that $AC : CB :: AH : HB$, and from the points C and H be inflected to any point of the circumference E two lines CE, HE meeting the same again in D and G; I say that $EC : CD :: EH : HG$.

PROP. IX.

P R O P. IX.

If in AB the diameter of a circle be taken any point C, and CD be drawn meeting the circumference in D and E, and from the point D be drawn DF perpendicular to CD, which meets the diameter AB in F and the circumference in G; then I say that $DC:CE::DF:FG$.

P R O P. X.

If in AB the diameter of a circle two points C and D be taken such that $AC:CB::AD:DB$, and through the centre E a perpendicular to AB be drawn, and from C a line be drawn to meet the same in F, and if through D any line DG be drawn to meet the circle in G and H, and from the point G be drawn GK the same side of DG as F is of the diameter AB to make the angle DGK equal to the angle CFE, and let the line GK meet the circle in L and the line CF in M; then I say that $GM:ML::GD:DH$.

P R O P. XI.

If from any point C in the diameter of a circle produced a perpendicular be raised and
from

from any point D in the same a line be drawn to cut the circle in E and F; then I say the rectangle EDF is equal to the rectangle ACB together with the square of CD.

P R O P. XII.

If from any point C in the diameter of a circle produced a perpendicular be raised and thereon CD be taken whose square is equal to the rectangle ACB, and CE be put equal CD, and from any point in DE as H a line be drawn to cut the circle in F and G; then I say twice the rectangle FHG is equal to the sum of the squares of HD and HE.

P R O P. XIII.

If in AB the diameter of a circle two points C and D be so taken that, C being without, and D either within or without the circle, the square of CD be equal to the rectangle ACB, and from C a perpendicular to AB erected, and any line drawn through D to cut the same in G and the circle in E and F; then I say the square of GD will be equal to the rectangle EGF.

The

The converse is also true, which is this.

If GC be perpendicular to AB the diameter of a circle and meets it without the circle in C, and if from G a line be drawn to cut the circle in E and F, and the diameter either within or without in D, and the square of GD be equal to the rectangle FGF; then I say the square of CD will be equal to the rectangle ACB.

P R O P. XIV.

Things remaining as in the last proposition, if the perpendiculars Eg and FH be demitted; then I say that the rectangle gCH is equal to the square of CD.

P R O P. XV.

If from C any point in the diameter of a circle AB produced a tangent be drawn, and from the point of contact D a perpendicular to the diameter DE be demitted; then I say that $AC : CB :: AE : EB$.

Or

Or conversely thus :

If in AB the diameter of a circle be taken two points C and E such that $AC:CB::AE:EB$, and from E a perpendicular ED raised, and CD drawn ; then I say CD touches the circle in D.

Or thus :

If in AB the diameter of a circle produced a point C be taken, and therefrom a tangent as CD be drawn, and in the diameter a point E be taken such that $AC : CB :: AE : EB$; then I say ED being drawn will be perpendicular to the diameter AB.

P R O P. XVI.

Let AB be any chord in a circle and CD another cutting the former in E, CB being joined, from D draw DF parallel to CB to meet AB in F ; I say that the rectangle AEF is equal to the square of DE.

P R O P. XVII.

If ABC be a triangle inscribed in a circle whose sides CA and CB are equal, and the rectangle

rectangle CBD equal to the square of AB, and let AE be any line cutting CB in F and the circle again in E, and from E let a parallel to AB be drawn to meet CB in G; then I say that the rectangle CFG : BF^2 :: CG : BD.

P R O P. XVIII.

Let ABC be a triangle inscribed in a circle, whose sides AB and AC are equal, and from A any line be drawn meeting the circle again in D and BC in E; I say that the rectangle DAE is equal to the square of AB.

P R O P. XIX.

Things remaining as in the last proposition, if lines touching the circle in A and C be drawn to meet in F, and FD be drawn cutting BC in G; I say that the rectangle BCG is equal to the square of CE.

P R O P. XX.

Let ABC be a triangle inscribed in a circle whose sides AB and AC are equal, and let AD be parallel to BC, and taking any point therein

C

D,

D, let the rectangle under AD and P be equal to the square of AB or AC, and from the points A and D let the lines AE, DE be inflected to any point E in the circle, meeting BC in F and G; I say the rectangle under FG and P = the rectangle BFC.

P R O P. XXI.

If in AB the diameter of a circle be taken two points C and D such that $AC:CB::AD:DB$, and D be within the circle, and DE be perpendicular to AB meeting the circle in E and F, and if through C any line be drawn meeting the circle in G and H, and the line DE in K, and GL touch the circle in G, and meet DE in L; then I say the rectangle LDK is equal to the square of DE.

P R O P. XXII.

If in AB the diameter of a circle be taken two points C and D such that $AC:CB::AD:DB$, and D be without the circle, and DE be perpendicular to AB, and through C be drawn any line meeting the circle in G and H, and
the

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the line DE in K, and GL touch the circle in G, and meet DE in L; then I say the rectangle LDK is equal to the rectangle ADB.

P R O P. XXIII.

If AB be the diameter of a circle and CD perpendicular thereto meeting it in C, and from the points A and B be inflected AE, BE to any point E in the circumference, meeting CD in F and G; I say the rectangle GCF is equal to the rectangle ACB.

P R O P. XXIV.

In AB the diameter of a circle let two points C and D be taken such that $AC:CB::AD:DB$, and the point D be within the circle, and DE be perpendicular to AB, meeting the circumference in E and F, and let through C any line be drawn meeting the same in G and H, and from the points G and H let GN, HN be inflected to any point in the same N, and let them meet DE in M and L; I say the rectangle LDM is equal to the square of DE.

P R O P. XXV.

Let AB be the diameter of a circle and CD perpendicular to the same meeting the circumference in C and D, and let E be the centre, and from C and D let CF, DF be inflected to any point F in the circumference meeting the diameter AB in G and H; I say the rectangle GEH is equal to the square of the radius AE.

P R O P. XXVI.

In AB the diameter of a circle let two points C and D be taken such that $AC:CB::AD:DB$, and let D be without the circle, and DE perpendicular to BD, through the point C let any line be drawn meeting the circumference in F and G, and from the points F and G let FH and GH be inflected to any point H in the circumference meeting DE in K and L; I say the rectangle KDL is equal to the rectangle ADB,

P R O P. XXVII.

In AB the diameter of a circle let be taken the point C, and CD be perpendicular to AB, meeting the circumference in D and N, in CD let

let be taken two points E and F on the same side of C such that the rectangle ECF may be equal to the square of CD, and from the points E and F let EG, FG be inflected to any point G in the circumference, meeting the same in H and K, and let HK when drawn meet the diameter AB in L; then I say that $AL : LB :: AC : CB$.

P R O P. XXVIII.

In AB the diameter of a circle produced let be taken the point C, and CD be perpendicular to AB, and therein be taken two points E and F on different sides of C such that the rectangle ECF, may be equal to the rectangle ACB, and from the points E and F let EG, FG be inflected to any point G in the circle, meeting the same in H and K, and let HK when drawn meet the diameter AB in L; then I say that $AL : LB :: AC : CB$.

P R O P. XXIX.

Let AB touch a circle in B, and any line AE be drawn equal to AB, and likewise from A
let

let any line be drawn to cut the circle in C and D, and let EC, ED be drawn meeting the circle again in F and G; then FG being drawn will be parallel to AE.

P R O P. XXX.

Let AB touch a circle in B, and therein be taken two points E and F on the same side of A such that the rectangle EAF may be equal to the square of AB, and from A let any line be drawn meeting the circle in C and D, and EC, FD be drawn meeting the circle again in G and H; then GH being drawn will be parallel to AB.

P R O P. XXXI.

Let AB touch a circle in B, and any line AE be drawn and therein be taken two points E and F on the same side of A such that the rectangle EAF may be equal to the square of AB, and from A any line be drawn to meet the circle in C and D, and EC, FD be drawn meeting the circle again in G and H; GH being drawn will be parallel to AE.

PROP. XXXII.

P R O P. XXXII.

Through any point A within a circle let a line be drawn meeting it in B and E, and therein two points F and G be taken such on different sides of A that the rectangle FAG may be equal to the rectangle BAF, and thro' A any line be drawn meeting the circle in C and D, and FC, GD being drawn to meet the circle again in H and K; then HK being drawn will be parallel to AB.

P R O P. XXXIII.

Let AB be a line without a circle, and from A and B two lines be drawn to touch the circle in C and D, and let the square of AB be equal to the sum of the squares of AC and BD, and from A any line be drawn to meet the circle in E and F, and BE, BF be drawn meeting the circle again in G and H; the points A, G, H, will be in a right line.

P R O P. XXXIV.

Let AB meet a circle in C and D, and A be without and B within the same, and let the rectangle

rectangle CAD be equal to the square of AB together with the rectangle CBD, and through A any line be drawn meeting the circle in E and F, and BE, BF be drawn meeting the circle again in G and H; then the points A, G, H, are in a right line.

P R O P. XXXV.

From the extremes of AB let two lines AC, BD be drawn to touch a circle in C and D, and in AB let a point E be taken on the same side of A with B such that the rectangle BAE may be equal to the square of AC, and also in AB another point F on the same side of B with E such that the rectangle EBF may be equal to the square of BD, and through A any line be drawn meeting the circle in G and H, and BG, BH be drawn meeting the circle again in K and L; then the points L, K, F, are in a right line.

P R O P. XXXVI.

If from A the vertex of a triangle ABC be drawn AD to any point D in the base, and DE be drawn parallel to AC, and DF to AB;

I say

I say the sum of the rectangles BAE, CAF will be equal to the square of AD together with the rectangle BDC.

P R O P. XXXVII.

Let A and B be two points in the diameter of a circle whose centre is C, and let the rectangle ACB be equal to the square of the semidiameter; bisect AB in D, and raise the perpendicular DM; from the point A draw AF to any point F in the circumference, and FE perpendicular to DM; then I say that the square of AF is equal to twice the rectangle contained by AC and FE.

P R O P. XXXVIII.

If any regular figure be circumscribed about a circle, and from any point within the figure there be drawn perpendiculars to all the sides of the figure; the sum of the perpendiculars will be equal to the multiple of the semidiameter of the circle by the number of the sides of the figure.

D

PROP. XXXIX.

P R O P. XXXIX.

Let there be any number of right lines intersecting in a point, and making all the angles about the point equal, and let any circle pass through the same point; I say the circumference thereof will be divided by the intersecting lines into as many equal parts as there are lines.

P R O P. XL.

If there be two triangles ABC, DEF, which have one angle A in one equal to one angle D in the other, and another angle B in the first equal to the sum of the angles D and E in the second; then shall the sides AC, BC, DE, EF be proportional.

P R O P. XLI.

The square of the line bisecting the vertical angle of any triangle is a mean proportional between the differences of the squares of each side including that angle, and the square of the adjacent segment of the base made thereby.

PROP. XLII.

P R O P. XLII.

If from the same point two tangents be drawn to a circle, and a line be drawn joining the points of contact, and another line to be intercepted between the tangents cut the foregoing which joins the point of contact, so as to be bisected in the point of intersection; then I say that the part of that line which is a chord of the circle will also be bisected by the same point.

And, conversely, if the chord cutting the line joining the points of contact be bisected by the point of intersection; then the continuation of the same to meet the tangents will also be bisected by the same point.

P R O P. XLIII.

If from one of the equal angles of an isosceles triangle a perpendicular be drawn to the opposite side; then I say that the rectangle contained under that side and it's segment intercepted by the perpendicular and the base is equal to half the square of the base.

P R O P. XLIV.

If in a line AB two points C and D be taken;
then I say that

$$\overline{AB + AD} \times BC + BC^2 = 2 ABC + BCD.$$

And moreover that

A C D B

$$\overline{AB + AD} \times CD + CD^2 = 2 ADC + BCD.$$

P R O P. XLV.

If from the vertical angle of any triangle two lines be drawn to make equal angles with the sides containing it, and to cut the base; then I say that the square of one side is to the square of the other side as the rectangle under the segments of the base contiguous to the first side is to the rectangle under the segments contiguous to the other side.

P R O P. XLVI.

If in AB the diameter of a semicircle any point C be taken, and from thence any line as CD drawn to meet the circumference in D, and a perpendicular DE be demitted; then I say that the square of the line AC is equal to the square

square of the line CD together with the rectangle under the sum of the distances of C from A and C from B and the line AE, when C is taken in the diameter AB produced; but equal to the square of CD together with the rectangle under the difference of the distances of C from A and C from B and the same line AE, when C is taken in the diameter itself.

P R O P. XLVII.

If from one angle A of a rectangle ABCD a line be drawn to cut the two opposite sides BC, DC, the former in F, and the latter produce in E; then I say that the rectangle EAF is equal to the sum of the rectangles EDC, CBF.

P R O P. XLVIII.

If a rectangle be inscribed in a right-angled triangle, so that one of its angles coincide with the angle of the triangle; then I say that the rectangle under the segments of the hypotenuse is equal to the sum of the rectangles under the segments of the sides about the right angle made by this inscription.

PROP. XLIX.

P R O P. XLIX.

If from the same point C two tangents be drawn to a femicircle whose diameter is AB, and if the extremes of the diameter and the points of contact be joined, either cross-ways by two lines intersecting in F, or other-ways by two lines intersecting in H; then I say that CF or HC produced to meet the diameter AB will be perpendicular to the same.

P R O P. L.

If in a femicircle whose diameter is AB the chord of 60° equal to the radius be inscribed and from the center E a perpendicular drawn thereto and produced to meet the circumference in F; then I say that AF, EF, BF are continual proportionals.

P R O P. LI.

If a line be cut in extreme and mean proportion; then I say that the square of the whole, the rectangle under the whole and the greater segment, and the rectangle under the whole and

and the leſſer ſegments are continual proportionals.

P R O P. LII.

If three lines are continual proportionals ; the ſum of the ſquares of the mean and the greater extreme is to the rectangle contained under the ſame, as the ſum of the extremes is to the mean.

P R O P. LIII.

In every right-angled triangle, as the hypothenuſe is to the ſum of the ſides about the right angle, ſo is the ſaid ſum, to the ſum of the hypothenuſe and twice the perpendicular from the right angle.

P R O P. LIV.

If a right line AD be any-ways cut in B, and from thence a perpendicular BE be erected equal to a mean proportional between the whole AD and the part AB, and a circle be drawn through the points A, D, E, and from A a per-

perpendicular be erected to meet the circumference in F; then I say that AF, AB, BE, AD are four continual proportionals.

P R O P. LV.

In every right-angled triangle, as the difference between the hypotenuse and one side is to the difference between the same side and its adjacent segment, so is the same side to the same segment.

P R O P. LVI.

If HC be a tangent to a circle meeting the diameter DB produced in H, and from the point of contact C a perpendicular CK to that diameter be drawn, and likewise a line from H cutting the circle in F and G, and the perpendicular CK in I, and E be the nearest point to H; then I say that the square of HF is to the square of the tangent HC as FI to IG.

P R O P. LVII.

If one side AC of an equilateral triangle ABC be produced to E so that CE may be equal to AC,

AC, and from A a perpendicular to AC raised, and from E a line drawn through the vertex B to meet the perpendicular in D; then I say that BD is equal to the radius of the circle which circumscribes the triangle.

P R O P. LVIII.

If BD bisect the vertical angle B of a triangle ABC and meet the base in D, and if with either of the other angular points A or C as center and the adjacent segment of the base as radius a circle be described to cut BD again in E; then I say that BE is to BD as that segment used as radius is to the other.

P R O P. LIX.

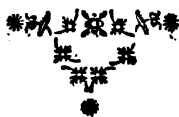
If BD bisect the vertical angle B of the triangle ABC, and if on BA or BC from B be put a third proportional to the other side and the bisecting line; then I say the rectangle under that side on which it is put and its remainder when the third proportional is taken from it is equal to the square of the adjacent

E segment

segment of the base made by the bisecting line.
i. e. $BCE = CD^2$, or $BAE = AD^2$.

P R O P. LX.

If an isoscles triangle be inscribed in a semicircle and one of the equal sides produced, and if from any point E in the diameter a perpendicular thereto be drawn to cut the side, the circle, and the side produced in the points G, H and F respectively; then I say that EG, EH and EF are continual proportionals.



GEOME-



GEOMETRICAL PROBLEMS.



P R O B. I.

THROUGH a given point A without a given circle to draw a line to cut the circumference in two points X and Y such, that the rectangle under the segments AX and XY may be equal to a given plane.

P R O B. II.

Two circles being given neither touching, nor cutting, nor including one the other; to find a point in the line joining their centers such that, tangents being drawn from thence to the circles, those tangents may obtain a given ratio.

P R O B. III.

Having a point C given in the diameter AB of a circle produced; it is required to find a point

E 2

point

point X in the continuation BC such that, a tangent being drawn therefrom, the said tangent may be equal to XC.

P R O B. IV.

A circle being given and two points either within or without it ; through these two points it is required to draw another circle which shall bisect the periphery of the former.

P R O B. V.

Having two points A and B given either both within or both without a given circle ; it is required to inflect therefrom two lines AX and BX to the same point of the circumference X such that, AX and BX meeting the circumference again in Y and Z, YZ may be parallel to AB.

P R O B. VI.

Having a circle given, and likewise three points D, E, F, being in a right line, and either all three without the circle, or D and E within and F without ; it is required from D
and

and E to inflect two lines DA and EA in such a manner to the same point A of the circumference, to meet it again in B and C, that BCF may be a right line.

P R O B. VII.

Four right lines being given, any three of which are greater than the remaining one ; it is required of these four lines to constitute a quadrilateral figure which will stand in a circle.

P R O B. VIII.

Having three points given in the circumference of a circle ; to draw a diameter of that circle such that, perpendiculars being let fall from the given points upon it, the segments of this diameter, intercepted by these perpendiculars, may bear a given ratio to each other.

P R O B. IX.

Having two points given in the circumference of a circle, to draw a diameter of that circle such that, perpendiculars being let fall from the given points upon it, the segment of
this

this diameter, intercepted by these perpendiculars, may be equal to a given line.

P R O B. X.

Having three points given in the circumference of a circle; to find a point from which three lines being drawn to the given points, these lines shall make angles with each other equal to two given angles.

P R O B. XI.

Having a point given within a circle, and suppose therefrom an indefinite number of lines drawn to the circumference, and upon each of these lines as bases isosceles triangles be constituted, one of whose equal sides is a tangent to the circle; it is required to determine the locus of the vertices of those isosceles triangles.

P R O B. XII.

In a given segment of a circle to find a point in the circumference such that, two lines being drawn from thence to the extremes of the base, the rectangle under these lines shall be equal to the square of their difference.

PROB. XIII.

P R O B. XIII.

To produce the diameter of a given circle to such a point that, a tangent being drawn from thence, and a perpendicular let fall from the point of contact upon the diameter, the rectangle under the tangent and such perpendicular shall be to the rectangle under the diameter and the lesser segment of the diameter made by the perpendicular in a given ratio.

P R O B. XIV.

To find a point in the diameter of a given semicircle, or in the diameter produced, such that, a line being drawn through it making a given angle with the diameter, and terminating in the circumference, the square of that line shall be to the rectangle under the parts of the diameter intercepted by it in a given ratio.

P R O B. XV.

Having a rhombus ABDC given; it is required from one of it's angles A to draw a line AG to meet one of the sides produced in G, so that the whole line AG may be to the part of it EF intercepted between the diagonal CB and the side BD in a given ratio.

PROB. XVI.

P R O B. XVI.

Having a triangle ABC given whose sides BA and BC are supposed indefinitely produced, and a point D given any where in the other side AC; it is required through D to draw a line IDH, intercepted between BA and BC, to cut off a triangle IBH equal to the original one ABC,

P R O B. XVII.

Having a semicircle given, and a point D in it's diameter AC; it is required through D to draw another semicircle to be included in the former of such a magnitude that, a tangent being drawn thereto from C as CEB, the part thereof EB, intercepted between the point of contact E and it's intersection with the former given semicircle B, may be equal to DA,

P R O B. XVIII.

To divide a circle into any number of parts which shall be as well equal in area as in circumference.—N. B. *This may seem a paradox, however it may be effected in a manner strictly geometrical.*

F I N I S.

